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ABSTRACT

GRADES OR AGES: Secondary. SUBJECT MATTER: Geometry. ORGANIZATION AND PHYSICAL APPEARANCE: The subject content of the guide is arranged in four columns--major areas, significant outcomes, observations and suggestions, references and films. The guide is mimeographed and spiral bound with a soft cover. OBJECTIVES AND ACTIVITIES: General objectives are listed in the introductory material, with more specific objectives in the significant outcomes columns. Activities are not listed in detail. INSTRUCTIONAL MATERIALS: Texts, films, and filmstrips are listed for the major areas, and there is a brief bibliography. STUDENT ASSESSMENT: A multiple choice test, with answers, is included to provide a means of evaluation. (MBM)

MATHEMATICS CURRICULUM GUIDE

GEOMETRY

Dr. Clarence E. Swingley, Acting Superintendent of Schools
Dr. Haron J. Battle, Assistant Superintendent-Instruction
Dr. Don T. Torreson, General Supervisor-Secondary Education
Dr. William H. Watson, General Supervisor-Curriculum Development
Mr. Paul Bohney, Mathematics Consultant

SCHOOL CITY OF GARY
Gary, Indiana

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Participants in the Preparation of this Guide

Mathematics Department Representative Committee

1966-67	1967-68
Ernest Bennett	Ernest Bennett
Paul Bohney	Paul Bohney
Doris Boose	Doris Boose
Lillian Bradley	Joseph Dolak
Robert Crowe	George Dunleavy
Joseph Dolak	Charles Hubbard
George Dunleavy	Elwood Lumpkin
Wilma Flewelling	Wayne Morgan
Charles Hubbard	John Pitts
Harold Jones	Maurice Smolinski
Wayne Morgan	Thomas Uecker
John Pitts	Ann Van Beek
Maurice Smolinski	
Thomas Uecker	

Geometry Curriculum Committee

Maurice Smolinske
James Yeary
Paul Bohney

Supervisory Personnel

Paul Bohney, Mathematics Consultant
William H. Watson, General Supervisor-Curriculum Development
Don T. Torreson, General Supervisor-Secondary Education

Secretarial Personnel

Sharon Marley

Procedure

The Department Representative Committee, working in their sessions, review the mathematics program and recommend specific improvements needed to strengthen instruction. Some of the recommendations require the service of special committees.

This supplement was proposed to up-date the geometry curriculum by the Department Representative Committee. The writing of the first draft took place during the summer of 1967. The first draft was submitted to the entire mathematics faculty for suggestions for incorporation in this edition. All materials were reviewed and edited by the Mathematics Consultant.

F O R E W O R D

Learning mathematics becomes more important to our society as technology advances. More people are needed to fill entry positions which require skill and knowledge of mathematics. Their skills and knowledge must be developed increasingly at higher levels and schools must help students strive toward this attainment. Well-prepared, energetic and enthusiastic mathematics teachers are primary contributors to these goals.

This mathematics guide represents another step in the on-going process of developing a strong mathematics curriculum in the Gary Public Schools. The guide outlines major areas of study in geometry, defines significant outcomes in behavioral terms, provides teaching suggestions, identifies learning materials, and includes evaluation items which can aid in determining students' achievement.

Appreciation is expressed to each individual, to the committee and supervisory personnel who have contributed to the development and the strengthening of the mathematics curriculum. We hope that the mathematics teachers will use this guide diligently, evaluate the results carefully, and share in identifying further revisions that will help keep the program moving forward in accord with the many worthwhile changes occurring in the field of mathematics.

CLARENCE E. SWINGLEY Acting Superintendent of Schools	HARON J. BATTLE Assistant Superintendent-Instruction

SCHOOL CITY OF GARY
Gary, Indiana

General Objectives of a Secondary Course in Geometry

In developing objectives for the course in geometry, the curriculum committee reviewed the experiences currently being given to students in the elementary grades and in the junior high school through an intuitive, experimental, and informal study of geometry. It was the view of the committee that these early experiences with geometry are not only of an enrichment nature, but also essential for an introduction to formal demonstration which to a great extent characterizes a secondary course in geometry. Experiences with geometric ideas of points, lines, planes, sets, intersecting and parallel lines and planes, skew lines, rays, angles, as well as plane and space figures along with some geometric constructions and notions about congruency, similarity, and measurement are vital prerequisites for a meaningful and worthwhile study of formal geometry in the secondary school. Secondary teachers can lend encouragement to elementary teachers in underlining the importance of this aspect of elementary school mathematics.

The general objectives of a secondary course in geometry set forth by the curriculum committee are as follows:

1. To develop certain basic geometric concepts.
2. To develop the ability to do deductive applications.
3. To develop a command of precise and concise language.
4. To encourage and increase ability to make conjectures.
5. To apply and extend algebraic skills and understandings.
6. To further knowledge of coordinate geometry.
7. To develop spatial perception through study of relationships among geometric elements.
8. To stimulate systematic and logical thought through orderly and neatly written work.

GENERAL TEACHING SUGGESTIONS

Most films available for teaching geometry can best be used as a summary of ideas that have been previously presented.

It is recognized that single concept film loops are currently being develo and their value will have to be determined from actual use by students.

The overhead projector is a valuable aid in presenting most topics of geometry. Not only its dramatic appeal, but also its conservation of time in presenting ideas makes this so. Every effort should be made to have overhead projectors available on a permanent basis in each geometry classroom.

Time can be well spent using various geometric models during classroom presentations. The student will also benefit from constructing models from cardboard and other appropriate materials.

While references other than the currently adopted textbook have been include in this guide, their major purpose is to give teachers suggestions on alternate approaches and methods. They may also be used as sources for other exercises or test items. When necessary, care must be taken to rewrite statements and symbols which agree with those in the currently adopted text.

Finally, it should be noted, in the spirit of this guide, that success of students in studying geometry can be related to adequate daily teacher preparation, including clear ideas of outcomes desired, interesting and enthusiastic presentations, and total involvement of students in learning situations.

GEOMETRY

MAJOR AREAS	SIGNIFICANT OUTCOMES
I. Basic Concepts of Geometry A. Sets 1. Representing 2. Relationships B. Real Numbers and the Number Line 1. Rational and irrational numbers 2. 1-1 correspondence and order relation 3. Distance between points C. Undefined Terms 1. Point 2. Line 3. Plane D. Definitions 1. Space a. sets of points b. line segments c. rays 2. Angles a. measurement b. classification c. special pairs d. bisector	I. 1. Specify sets by roster and rule methods. 2. State whether a set is finite or infinite, and if finite, state whether null or not. 3. Identify subsets and union or intersection of given sets. 4. Use Venn diagrams to interpret set relations. 5. State whether numbers are rational or irrational. 6. Portray a one-to-one correspondence between two sets. 7. Write ordering symbols which produce true statements about coordinates on a line. 8. Compute distance between points on a number line. 9. Use the definition of absolute value to determine the solution of open sentences. 10. State whether or not conditions given are valid examples of points, lines, planes and relationships such as "contains", "on", "passes through", "intersection". 11. Draw figures which portray relationships of points, lines and planes. 12. Identify a term with its definition. 13. Use a protractor to measure and draw angles. 14. Compute with denominate measures of angles.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
I. This is a review of ideas which have been introduced intuitively in earlier grades. The teacher's major emphasis in this area should be to develop the preciseness of definitions necessary to enable students to think about geometry from a more abstract viewpoint.	I. 4 Chapter 1* 1 Chapter 2,3 2 Chapter 1 3 Chapter 1,3 6 Chapter 2 7 Chapter 1,6,7 8 Chapter 2,3,4 11 Chapter 1,2,3
The teacher might emphasize the following qualities of a good definition: 1. The term to be defined should be placed in its nearest class. 2. The necessary distinguishing properties should be given. 3. The definition should be reversible. 4. The definition should involve previously defined terms.	Films: Angles. (KB) B&W, 10 min.
It is necessary at this beginning point that students experience success. Their apprehension about extreme difficulty in the study of geometry can thereby be removed.	
A student cannot ignore any facet of the information presented in this topic because it is all needed to comprehend geometry.	
	*The first reference listed is the currently adopted text.

MAJOR AREAS	SIGNIFICANT OUTCOMES
II. Inductive and Deductive Reasoning A. Intuition - Induction 1. Triangles a. definitions b. intuitive and inductive experiences 2. Polygons a. definitions and nomenclature b. intuitive and inductive experiences 3. Quadrilaterals a. definition b. intuitive and inductive experiences 4. Circle and Spheres a. definitions b. intuitive and inductive experiences c. relationships with other figures B. Deduction and Proof 1. General experiences a. comparison with induction b. if - then statements 2. Characteristics of definitions and postulates 3. Postulates a. real number properties b. equality properties c. properties of inequalities d. points, lines, and planes 4. Proof of theorems and exercises	II. 1. State whether a specific conclusion has been reached by intuition, inductive or deductive reasoning. 2. From a set of facts state inductive or deductive conclusions. 3. Name and measure parts of a triangle and state relationships about the sum of the measures of the angles and about the sum of the lengths of two sides compared to the length of the third side. 4. Classify triangles by angle and side measurements. 5. Derive conclusions about polygons by induction and apply these conclusions to other polygons and test the results for accuracy. 6. Classify quadrilaterals according to given conditions. 7. Name lines and segments as they relate to a circle. 8. Draw figures to illustrate and derive conclusions from concentric circles, tangent circles, common tangent, circumscribed and inscribed figures. 9. State hypothesis and conclusions of a given statement. 10. State whether or not a given definition is acceptable. 11. State which properties of the real numbers are illustrated in given equalities and inequalities. 12. Determine validity of statements by use of postulates and theorems. 13. Write two column deductive proofs including those which require the use of the property of betweenness and other properties of real numbers.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
II. This is the students' first introduction to formal proof. Section B of this major area must be introduced with much student-teacher verbal interaction. The understanding of what constitutes a proof must be well-founded in each student. Wide variation in students' rate of comprehension of this idea is present in every class.	II. 4 Chapter 2, 3 1 Chapter 1, 4 2 Chapter 1, 13 3 Chapter 2, 4 6 Chapter 6 7 Chapter 3 8 Chapter 1, 2 11 Chapter 3, 5
Current mathematical literature shows a preference for "postulates" rather than "axioms."	Films: <u>Triangles: Types and Uses.</u> (Cor) B&W, 11 min. <u>Polygons.</u> (KB) B&W, 10 min. <u>Quadrilaterals.</u> (KB) B&W, 10 min. <u>The Circle.</u> (KB) B&W, 10 min.
The inductive reasoning discussed in this major area is not mathematical induction based on the well-ordering principle of the positive integers.	Filmstrips: MG-542-5 <u>Proof</u> (SVE)
By examining concepts considered in this major area, it was determined that students must either have from their past experiences or must learn a minimum of ninety-nine vocabulary words.	
Occasional matching quizzes of terms with definition and spelling quizzes may indicate areas in which special attention should be given.	
Emphasize that the student should not rely on the figure for reasons in a proof. Some optical illusions might convince students of this.	

MAJOR AREAS	SIGNIFICANT OUTCOMES
III. Angles	III. 1. Define an angle.
A. Angle-Measurement 1. Postulates 2. Betweenness of rays 3. Angle-addition theorem 4. Uniqueness of angle measure and bisector	2. State the difference between equality of angles and equality of measures of angles. 3. Recognize instances where the angle-addition theorem justifies conclusions.
B. Theorems about Angles 1. Straight angle and right angle equalities 2. Conditions pertaining to perpendicular lines 3. Supplementary and complementary angles 4. Vertical angles	4. Write proofs requiring the use of the angle-addition theorem. 5. State the relationship between perpendicular lines and right angles.
C. Requirements for Formal Demonstration of Theorem 1. Statement 2. Figure 3. Given 4. To prove 5. Analysis 6. Proof	6. From given statements and an associated drawing, recognize right angles, perpendicular lines, adjacent angles, vertical angles, opposite rays, complementary and supplementary angles by indicating the truth or falsity of statements or by response to oral questions. 7. Express the measure of an angle when the measure of its complement or supplement is given as an algebraic expression. 8. Express the measure of complementary or supplementary angles when the ratio of their measures is given. 9. State in order six essential parts of a demonstration of a theorem. 10. Given a statement, draw a figure, list given conditions based upon the figure drawn, and list conclusions to be proved in terms of the figure and the given statement.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
III. The teacher should note with care the limitations placed upon the measure of an angle in the text being used. The text by Jurgensen, et. al., places these limits greater than zero and less than or equal to 180, while the text by Moise and Downs specifies the open interval between zero and 180. Direct students to draw an angle as follows: Have students locate a vertex and choose two points on the half circle having the vertex as center. As the student draws the rays from the vertex to the chosen points on the half-circle emphasize that the rays represent sets of points and that the angle formed consists of the union of these points. This exercise can be extended to include the concept of angle measure. The student might profit from measuring an angle using two or more protractors of different radii. The rationale for these postulates of measured angles rests on the definition that angles consist of two distinct rays. Students can profit from the knowledge, that in studying trigonometry, angles will be redefined to include angle measure of all real numbers. Students can profit from many exercises which require the first four essential parts of a demonstration of a theorem; namely, statement, figure, given and to prove. To further acquaint students with reasoning patterns, a short study of elements of logic is desirable. The appendix of (4) may be used.	III. 4 Chapter 4 1 Chapter 3, 1 2 Chapter 1 3 Chapter 1 6 Chapter 4, 5 7 Chapter 2, 4 8 Chapter 4 11 Chapter 1, 5, 9 Filmstrips: MG-542-3 <u>Angles</u> (SVE)

MAJOR AREAS	SIGNIFICANT OUTCOMES
IV. Parallel Lines and Planes A. Basic Properties B. Angles Formed by Transversals 1. Definitions a. transversals b. corresponding angles c. alternate interior angles d. alternate exterior angles 2. Equal pairs 3. Perpendicular transversals C. Indirect Proof 1. Negative of a statement 2. Arriving at a contradiction D. Proving Lines Parallel 1. Parallel postulate 2. Induction or conjecture of the converse of postulates and theorems. 3. Use of auxiliary lines E. Applications to Angle Measure in Triangles 1. Sum of the measures of angles 2. Corollaries	IV. 1. Give examples of skew, parallel, and intersecting lines and of parallel and intersecting planes from the classroom and from a figure. 2. Identify alternate interior, alternate exterior, and corresponding angles. 3. Compute the measure of angles formed by parallel lines cut by transversals when given the measure of one angle. 4. Prove angle relationships. 5. State the negative of statements. 6. State the converse of statements. 7. Determine the validity of a statement and its converse. 8. Write an indirect proof in argumentative form. 9. State conditions for lines being parallel. 10. Prove lines parallel. 11. Compute the measures of angles of triangles using theorems either about the sum of the interior angles of a triangle or about the equality of an exterior angle of a triangle to the sum of its two remote interior angles.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS

IV. While alternate interior angles generally are studied in detail, the matter of alternate exterior angles in this major area is often slighted. An excellent opportunity is available to show students that additional theorems are possible beyond those listed in some textbooks. For example consider:

"If two parallel lines are cut by a transversal, alternate exterior angles are equal", and the converse of this theorem.

Students can prove these theorems and have them available for use throughout the remainder of the course and thereby develop a better idea of a converse.

Students should be cautioned regarding the improper writing of "AB||CD" where AB and CD represent measure of line segments.

The teacher might prefer to approach indirect proof through the concept of a contrapositive.

Special attention is given to the following postulate: "If two parallel lines are cut by a transversal, corresponding angles are equal." It is recommended that it be interpreted to emphasize the condition of parallelism. The same should be done in theorems and converses related to this postulate. For example, this might be reworded by the teacher:

"When two lines are cut by a transversal, if they are parallel, corresponding angles are equal."

Possible symbols:

Segment ⟶ $\overline{AB}$

Measure of Segment ⟶ |AB|

Line ⟶ AB

IV. 4 Chapter 5
1 Chapter 4, 5, 6, 9
2 Chapter 3, 4, 13
3 Chapter 5
6 Chapter 3, 9, 10
7 Chapter 4, 7, 10
8 Chapter 6
9 Chapter 9
11 Chapter 5, 7, 17

Films:

<u>Properties of Triangles.</u> (KB) B&W, 10 min.
<u>Parallel Lines.</u> (IV) B&W, 10 min.

Filmstrips:

1148 - <u>Sum of the Measures of Angles of a Triangle</u> (FOM)
1113 - <u>Parallelograms and their Properties</u> (FOM)
1139 - <u>The Parallel Postulate</u> (FOM)
1140 - <u>Angle Sums for Polygons</u> (FOM)

MAJOR AREAS	SIGNIFICANT OUTCOMES
V. Congruent Triangles and the Properties of Quadrilaterals	V. 1. Pair corresponding parts of two triangles.
A. Triangle Congruency 1. Definitions 2. Conditions	2. Write and interpret congruency of parts.
a. corresponding parts b. postulates, theorems, and corollaries 3. Proofs	3. Select postulates, theorems, and corollaries necessary to prove congruency of triangles.
a. non-overlapping triangles b. overlapping triangles c. equality of corresponding parts	4. Write proofs of triangle congruency.
d. limitations upon auxiliary lines e. special triangles	5. Redraw overlapping triangles so that they appear as separate triangles.
B. Properties of Quadrilaterals 1. Distance definitions	6. List all corresponding parts of triangles proved congruent.
a. from a point to a line or plane b. between parallel lines or planes	7. Write proofs of segment or angle relationships which require triangle congruency.
c. from a point to a figure 2. Special quadrilaterals a. angles and segment relations	8. State whether or not conditions for an auxiliary line determine the line.
b. conditions for proof that quadrilaterals are special	9. Write proofs requiring the use of auxiliary lines.
	10. Prove and use properties of special quadrilaterals.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
V. While congruency is a special case of similarity, it is normally treated before similarity.. It is suggested that the teacher follow this practice. Similarity is a more difficult idea for the student to apply and congruency gives him some previous and easier experiences which aid him in developing concepts for application to two or more figures. It is recommended that the designation of congruency of triangles be written such that corresponding vertices are arranged in order of their correspondence. This enables students to select corresponding parts from the congruency statement as well as from the figure. This would help students avoid errors in the selection of the correspondences. While many teachers may be reminded of the idea of superposition in the discussion of congruency, it is hoped that the postulation of congruency will be much more meaningful. The full meaning of congruency must extend to solid figures as well as plane figures --a case at point where superimposition would be impossible. The student should fully understand that congruency embodies the fact that any or all parts of a one-to-one correspondence between two figures have exactly the same size and shape. The properties of quadrilaterals are treated very briefly by some texts. These properties can very profitably be treated as a separate short unit of study. Many of the exercises offered by texts could be treated as theorems by the class since they will become useful in later work.	V. 4 Chapter 6 1 Chapter 6, 7, 8 2 Chapter 2, 4 3 Chapter 2, 8 6 Chapter 5, 6, 11 7 Chapter 6, 9 8 Chapter 5, Appendix VI 9 Chapter 8 11 Chapter 4, 7 Filmstrips: 1122 - <u>Congruent Triangles</u> (FOM)

MAJOR AREAS	SIGNIFICANT OUTCOMES
VI. Similarity of Polygons A. Ratio and Proportion 1. Definition of ratio 2. Definition of proportion a. terms b. means c. extremes 3. Algebraic applications B. Special Properties of a Proportion 1. Means-extremes product property 2. Equivalent-form property 3. Denominator-addition property 4. Denominator-subtraction property 5. Numerator-denominator sum property of equal ratios C. Similar Polygons 1. Definitions 2. Ratio of perimeters D. Similar Triangles 1. AA postulate 2. Corollaries 3. Applications to theorems about one triangle a. line parallel to a side and intersecting two sides of a triangle b. angle bisector 4. Corollary on parallel lines and transversals 5. Corresponding altitudes E. Properties leading to the proof of the Pythagorean Theorem 1. Definitions a. mean proportional b. projections 2. Properties determined by the altitude to the hypotenuse F. Pythagorean Theorem 1. Proof 2. Converse theorem 3. 30-60-90 triangle 4. Isosceles right triangle 5. Applications in solid figures 6. Projections	VI. 1. Write ratios in simplest and different forms. 2. Use ratios of three or more numbers to compute measures. 3. Use the means-extremes product property and its converse to write equivalent equations. 4. Solve proportions for specified variables. 5. Compute measures of sides and perimeters of similar polygons. 6. State conclusions based upon similar triangles and other polygons. 7. Prove triangles similar. 8. Compute measures of segments as determined by a line parallel to one side of a triangle. 9. Find measures of segments formed on transversals intersected by parallel lines. 10. Find the point of intersection of the bisector of an angle of a triangle with the opposite side by using the proportional properties of the angle bisector. 11. Compute the measure of an altitude in one of two similar triangles. 12. Write a statement about the similarity of the three triangles present when the altitude is drawn to the hypotenuse of a right triangle. 13. Compute measures of segments of a right triangle when the altitude is drawn to the hypotenuse. 14. Describe the projection of a figure on a line or plane. 15. Compute the measure of the projection of a given segment. 16. Use the Pythagorean theorem to compute the measure of a side of a right triangle. 17. Given the measure of three sides of a triangle, determine if it is a right triangle. 18. Given the measure of one side of a 30-60-90 or a 45-45-90 triangle, state the measure of the other two sides.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
VI. A ratio is the quotient of two numbers. It must be clear to the student that if the discussion is centered around segments for comparison, that the comparison is always between their measure. To keep students from making the invalid conclusion that polygons are similar if their corresponding angles are equal, use several physical drawings to demonstrate otherwise; i.e. a square and rectangle. It should be kept in mind, however, that drawings are used only to expedite or enhance understanding and constitute no condition for accepted validity. It should be emphasized that the properties of a proportion are all consequences of the properties of equality and are not assumed properties of a proportion. As with congruency, in writing statements about similarity, it is helpful to write vertices in order of correspondence. Since the length of a segment cannot be zero, division by zero in teaching ratio and proportion does not constitute a problem when applied to the meaning of segments of similar polygons. The teacher, however, should point out that making either or both denominators zero is equivalent to division by zero.	VI. 4 Chapter 7 1 Chapter 15, 16 2 Chapter 7 3 Chapter 6 6 Chapter 12 7 Chapter 8, 13, 14, 16 8 Chapter 7 11 Chapter 8, 12, 13 Films: <u>Similarity</u>. (M-H) Color, 14 min. Filmstrips: 1106 - <u>The Pythagorean Theorem</u> (FOM) 1116 - <u>Similar Triangles - Experiment and Deduction</u> (FOM)
VI. <u>SIGNIFICANT OUTCOMES</u> (continued) 19. Compute the measure of the diagonal of a rectangular solid and the measure of segments of regular pyramids.	

MAJOR AREAS	SIGNIFICANT OUTCOMES
VII. Trigonometry A. Definitions of trigonometric functions 1. Tangent 2. Sine 3. Cosine B. Table of Functions C. Applications 1. Angle of elevation 2. Angle of depression 3. Law of sines	VII. 1. State trigonometric ratios in terms of the sides of the right triangle or in terms of the unit circle. 2. Read values of trigonometric functions from a table. 3. Given the value of a trigonometric function determine the measure of the angle to the nearest integer. 4. Compute measures of segments of right triangles using trigonometric ratios. 5. Compute measures of acute angles of right triangles. 6. Use measures of angles of elevation or depression. 7. Compute measures of angles or sides in acute triangles using the law of sines.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
VII. In most geometry texts, the algebraic study of trigonometry is largely by implication. But the concepts driven home here may very well affect the student's desire to pursue mathematics on a higher level -- positively or negatively. The introduction of trigonometry should lead to a deeper appreciation of the importance of the study of similar triangles. However, the teacher might very well wish to widen the scope of algebraic application through the use of the unit circle. The coordinates in the plane which are sets of points on the circle can be shown to be such that (x,y)=(cosine a, sine a) where a is the measure of the angle at the center of the circle. This approach can lead to the fundamental trigonometric identity $\sin^2 a + \cos^2 a = 1$. Here the Theorem of Pythagoras has new application and new powers for the alert student. A further justification for introducing the circle is the later consequence of periodic functions. It is expected this would be a brief introduction. The Law of Sines and especially the Law of Cosines become useful to the study of physics and the physical properties of chemistry. The Law of Cosines has not been included in the outline of this major area. However, the teacher should feel free to present this idea, either to the entire class or as supplementary material to better students. Since understanding of the concept of function is of such importance in mathematics, the teacher might use this term at this time. For example, the tangent function could be defined as the set (a, tan a) such that tan a is defined as the ratio of the length of the opposite leg to the length of the adjacent leg.	VII. 4 Chapter 8 1 Chapter 16 2 Chapter 8 3 Chapter 13 6 Chapter 12 7 Chapter 16 9 Appendix X 11 Chapter 15 Films: Trigonometry.(EBF) B&W, 30 min. Filmstrips: 1108 - Indirect Measurement: Tangent Ratio (FOM) 1143 - The Law of Sines (FOM)

MAJOR AREAS	SIGNIFICIANT OUTCOMES
VIII. Circles A. Definitions 1. Equal circles 2. Central angle 3. Arcs 4. Measure of arcs B. Arc-Addition Postulate C. Angle Measure in Relation to Arc Measure 1. Inscribed angle 2. Angles formed by a secant and tangent rays a. vertex on the circle b. vertex in the interior c. vertex in the exterior D. Lines and Segments 1. Bisectors of arcs 2. Diameter perpendicular to a chord 3. Length of segments a. intersecting chords b. two secants from an external point c. a tangent and a secant from an external point	VIII. 1. Use proper names and symbols to describe lines, segments and arcs in relationship to a circle. 2. Identify the intercepted arc or arcs for various angles. 3. Compute the measures of angles from their intercepted arcs. 4. Compute the measure of arcs from the measure of angles which intercept them. 5. State the equality of central angles or inscribed angles intercepting equal arcs. 6. State the cases where inscribed angles are right angles, acute angles and obtuse angles, in terms of their intercepted arcs. 7. State whether a circle can be circumscribed about a quadrilateral when the measure of its angles are given. 8. Draw conclusions about the relationships between the tangent, point of tangency and the perpendicular diameter. 9. Recognize where equality exists when arcs are formed by parallel lines. 10. Draw conclusions about the relationships between the diameter perpendicular to a chord and its intersection with the arcs of the chord. 11. State conclusions about comparative distances of chords from the center of a circle. 12. Compute measures of segments of chords, secants, or tangents formed by intersections with one another and with a circle.

OBSERVATIONS AND SUGGESTIONS

VIII. The formal definition of a circle should be fully and readily comprehensive to every student in the class. The set of points in a circle could be illustrated by the motion, in a plane, of one end point of a segment where other end point is fixed.

It should be emphasized that a statement such as $\angle AOB = \widehat{AB}$ is ridiculous in terms of the definitions of an angle and an arc. Therefore, this type of statement is actually pertaining to their measures and not to their sets of points.

The theorem, "The measure of an angle formed by a secant ray and a tangent ray drawn from a point in a circle is one-half the measure of the intercepted arc", may be presented by an intuitive use of limits based upon the measure of an inscribed angle. However, it would be well to also establish this theorem by a formal deductive proof. This would help convince the student of the validity of the limit argument. Since this theorem occurs relatively early in the outline, its usual proof by deduction can not be done. It can be proved if the theorem, "A diameter drawn to a tangent at the point of tangency is perpendicular to the tangent", is available.* This can be proved by use of an indirect proof based on the definition of the distance from a point to a line.**

The outline of this major area contains the mimimum number of essential theorems. Many other important relationships should be covered as exercises and may, if the teacher so desires, be treated as theorems. Included among these are tangent and diameter relationships, equal arcs intercepted by parallel lines, radius and chord relationships, and comparative distances of chords from the center of the circle.

REFERENCES AND FILMS

VIII.
4 Chapter 9
1 Chapter 11, 12
2 Chapter 6
3 Chapter 9
6 Chapter 14, 15, 16
7 Chapter 2, 7, 10, 11, 12, 13, 15
9 Chapter 12
11 Chapter 4, 9, 10, 11

Films:

<u>Chords and Tangents of Circles.</u> (KB) B&W, 10 min.
*** <u>Dynamics of the Circle.</u> (MH) Color, 14 min.

Filmstrips:

1151 - <u>Diameter Perpendicular to a Chord</u> (FOM)
1123 - <u>Arc and Angle Measurement</u> (FOM)

*** To be available February, 1968.

* See Smith and Ulrich p. 266
** See Moise and Downs p. 426

MAJOR AREAS	SIGNIFICANT OUTCOMES
IX. Constructions and Loci	IX. 1. Differentiate between a drawing and a construction.
A. Constructions 1. Definitions a. drawing b. construction	2. Construct an angle equal to a given angle.
2. Basic constructions a. an angle equal to a given angle with a given ray as side	3. Construct an angle bisector. 4. Construct perpendicular lines.
b. angle bisector c. perpendicular bisector of a segment d. perpendicular line to a given line through a given point	5. Construct angles of measure 30, 45, 60, etc. 6. Construct a line parallel to a given line by at least two methods.
e. a line parallel to a given line through a given point not on the line.	7. Construct a tangent to a circle at or from a given point.
3. Other constructions a. bisector of a given arc b. determination of the center of an arc or of a circle	8. Circumscribe a circle about a triangle or a polygon if possible.
c. tangents to a given circle d. circumscribed and inscribed circles	9. Inscribe a circle in a triangle or a polygon if possible.
e. dividing a segment into any number of equal parts f. segments of proportional measure	10. Divide a segment into two, three, four, or five equal segments. 11. Construct a fourth proportional and a mean proportional.
B. Locus of Points 1. Definition 2. Fundamental loci a. at a given distance from a given point	12. Illustrate and describe the locus of points fitting fundamental loci.
b. at a given distance from a fixed line	13. Illustrate and describe the intersection of loci.
c. equidistant from two parallel lines d. equidistant from two points e. equidistant from the sides of an angle f. equidistant from two intersecting lines g. vertex of the right angle of a right triangle with a given hypotenuse 3. Intersection of loci 4. Loci in the development of a construction (optional or enrichment)	

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
IX. The extent to which the teacher may choose to emphasize part A of this unit will be determined by whether the emphasis earlier in the course has been on drawing or on construction of figures. If constructions have been taught earlier, concentration on the proof of the validity of these methods will be in order. The order of the construction of perpendiculars in this outline may be contrary to some texts. However, if the method of perpendicular bisecting of a segment is taught first, the other two perpendicular constructions can be taught as a special case of the perpendicular bisector where the end-points of the segment are to be determined as the initial step. The methods for circumscribing a circle about a triangle and inscribing a circle in a triangle may be extended to polygons in general. If the perpendicular bisector of the sides of any polygon are all concurrent, a circle can be circumscribed about it. If the bisectors of all of the angles of any polygon are concurrent, a circle may be inscribed in it. The teacher can emphasize that triangles and regular polygons have both an in-center and a circumcenter. The fundamental loci should be described by two different sets of points, depending upon whether the conditions are to be met in a plane or in space. The similarity between the locus of points equidistant from the sides of an angle and the locus of points equidistant from two intersecting lines justifies the treatment of the first as a special case of the second.	IX. 4 Chapter 10 1 Chapter 5, 12, 14 2 Chapter 2, 3, 6, 7, 10, 11 3 Chapter 1, 2, 3, 7, 8 6 Chapter 4, 8, 15 7 Chapter 2, 7, 10, 11, Appendix 9 Appendix XII 11 Chapter 1, 6, 11, 17 Film: <u>Locus.</u> (MH) Color, 13 min.

GEOMETRY

MAJOR AREAS	SIGNIFICANT OUTCOMES
X. Ordered Pair and Ordered Triples	X. 1. Draw graphs representing sets of numbers.
A. Correspondance between Points and Numbers 1. On a line 2. In a plane 3. In a space	2. Identify corresponding specified sets and graphs. 3. Draw graphs of solution sets for equalities and inequalities on the Cartesian plane.
B. Symmetry 1. With respect to a point 2. With respect to a line 3. With respect to a plane	4. Find coordinates of points symmetric with respect to the origin, x-axis, and y-axis.
C. Graphs and Formulas of Algebraic Conditions 1. Open sentence of one variable a. intersection b. union 2. Distance between two points 3. Circle 4. Midpoint of a segment	5. Compute the distance between two points on the coordinate plane and in coordinate space. 6. State the center and radius of a circle from its equations. 7. Write the equation of a circle when given its center and radius.
D. Lines 1. Slope a. formulas b. parallel lines c. perpendicular lines 2. Linear equations a. $Ax+By=C$ b. $y-y_1=m(x-x_1)$ 3. Graphing by the two intercepts 4. Solution of intersecting lines	8. Find the midpoint of a segment when given the coordinate of its endpoints. 9. Find the slope of the line determined by two points. 10. State readily whether the slope of an observed line is positive, negative, zero, or undefined.
E. Proofs 1. Choice of axes 2. Lines, segments, and points 3. Polygons	11. Determine by their slopes whether lines are perpendicular or parallel. 12. Write equations of lines by using the point-slope form. 13. Given the point-slope form of the equation of a line, draw its graph. 14. Given the equations of two lines, compute the intersection 15. Place geometric figures on co-ordinate axes to the best advantage.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS

X. The student's concept of symmetry can be tested by the teacher posing questions about the sphere, hemisphere, tangent spheres and tangent hemispheres. Can he find a point (line or plane) with respect to which a sphere has symmetry? Can he do the same for a hemisphere, for two tangent spheres? What condition is required before two tangent hemispheres have symmetry with respect to a line or plane? The obvious answer to this last question is when the bases are parallel, but symmetry also occurs when the bases form equal dihedral angles with the tangent plane.

Up to the present, algebraic applications have been limited to the solution of problems involving measures of geometric figures. This major area lends itself to the feasibility as well as to the advantages of expanding algebraic applications to geometric principles. Algebraic manipulations may often be simplified by the choice of geometric interpretation. Whether or not time can be spent on developing the ability to write proofs in coordinate geometry will have to be determined by the time left for comprehensive treatment of the areas which follow.

X. <u>SIGNIFICANT OUTCOMES</u> (continued)

16. Compute the coordinates of points of tri-section of a given segment.

X. 4 Chapter 11, 12
1 Chapter 20
2 Chapter 11
3 Chapter 3, 7, 8, 9
6 Chapter 13, 15
7 Chapter 7, 8, 9, 10
8 Chapter 3
9 Chapter 8, 9
11 Chapter 17

Films:

Graphing Linear Equations. (Coronet) B&W, 11 min.

Filmstrips:

1107 - *An Introduction to Coordinate Geometry* (FOM)
1115 - *The Slope of a Line* (FOM)
1124 - *Geometric Proof Using Coordinates* (FOM)
1132 - *Perpendicular Lines in Coordinate Geometry* (FOM)

GEOMETRY

MAJOR AREAS	SIGNIFICANT OUTCOMES

MAJOR AREAS

XI. Area of Geometric Regions

A. Polygons
  1. Definitions
     a. polygonal region
     b. area
  2. Postulates
     a. congruent triangles have equal areas
     b. area-addition
     c. area of a rectangle
  3. Area theorems and corollaries
     a. square
     b. parallelogram
     c. triangle
     d. trapezoid
  4. Similar regions
     a. triangle
     b. polygons

B. Regular Polygons
  1. Special properties
     a. a circle can be circumscribed about any regular polygon
     b. a circle can be inscribed in any regular polygon
  2. Definitions
     a. center
     b. radius
     c. apothem
     d. central angle
  3. A central angle of a regular n-gon has measure of 360/n
  4. Area formula
  5. Ratio of the areas of similar polygons

C. Circles
  1. Circumference
  2. Area
  3. Length of an arc
  4. Sectors and segments
     a. definitions
     b. areas

D. Area Constructions

SIGNIFICANT OUTCOMES

XI.
1. State when the area-addition postulate is applicable.
2. Compute areas of polygons.
3. Use properties of proportion to compare the measures of similar polygons.
4. Compute the measure of a central angle of a regular polygon.
5. Compute the radius and apothem for an equilateral triangle, square, and regular hexagon.
6. State the definition for π.
7. Recognize that π is irrational.
8. State the decimal approximation for π to five significant digits.
9. Compute the circumference and area of circles of given radii or diameter.
10. Given the circumference or area of a circle, compute the radius.
11. Compute the measure of an arc when given its measure and radius.
12. Compute the measure of an arc when given its length and radius.
13. Compute the area of a sector.
14. Compute the central angle for a sector.
15. Construct the square with area equal to the sum of the areas of two given squares.

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
XI. This major area serves as the foundation to many areas of application not necessarily purely mathematical. Some principles will find their way into physics, chemistry, or other related fields and the emphasis must be on understanding rather than memorization. The student should understand that the computation of area involves the product of one or more pairs of perpendicular segments. Proofs of more advanced formulas rest on this basic idea. A practice that is sometimes used is to write 6 sq. in. as 6 in.^{2} This practice can be confusing to the student, as he may interpret 6 in.^{2} as 6 inches square. It may not be obvious to the student that a figure of 6 square inches and one 6 inches square are not the same. In most of the exercises about the circle, answers should be left in terms of π to increase the students recognition of this symbol as a number. The teacher should stress that the numerical values used for π are approximations. Two circles can be thought of as being similar figures in the sense of having the same shape. Therefore, the properties of similarity concerning lengths of corresponding segments, circumferences, and areas will also apply to two circles.	XI. 4 Chapter 13 1 Chapter 17, 18 2 Chapter 9, 10 3 Chapter 9, 10 6 Chapter 11, 16 7 Chapter 14, 15, Appendix 9 Chapter 11, 12 11 Chapter 8, 14 Films: The Meaning of Pi. (Coronet) B&W, 10 min. Areas. (KB) B&W, 10 min. The Meaning of Area. (MH) Color, 14 min.

GEOMETRY

MAJOR AREAS	SIGNIFICANT OUTCOMES
XII. Measures of Solids	XII. 1. Associate names of space figures with drawings and models.
A. Prisms and Pyramids 1. Definitions 2. Prisms a. Lateral area b. Total area c. Volume d. Diagonal of a right rectangular prism	2. List vertices, faces, and edges of given polyhedrons. 3. Compute lateral areas, total areas and volumes of prisms, pyramids, cylinders and cones.
B. Cylinders and Areas 1. Definitions 2. Lateral area 3. Total area 4. Volume	4. Compute the area and volume of a sphere. 5. Compute the diagonal of a right rectangular prism.
C. Sphere 1. Area 2. Volume	6. Compute corresponding measures of similar solids.
D. Similar Solids 1. Ratios of corresponding segments 2. Ratios of areas of corresponding surfaces 3. Ratios of volumes	

OBSERVATIONS AND SUGGESTIONS	REFERENCES AND FILMS
XII. A cross-check of mathematical writers shows much disparity between definitions for the term geometric solid. Some definitions include the interior points in union with the surface points, while others exclude the interior points altogether. At least one author defines a sphere in the second manner and other solids in the first. The consensus, as formed by studying several definitions, seems to favor the inclusion of the interior points. A statement which seems to define a solid well, according to this viewpoint, is: "If a set of points separates space into two distinct regions, one of which is finite in extent, the union of this separation set of points and the set of points of the finite region is a solid."	XII. 4 Chapter 14 1 Chapter 13, 19 2 Chapter 2, 4, 12 3 Chapter 8, 9, 10, 11 6 Chapter 17 7 Chapter 9, 17 9 Chapter 11, Appendix VII 11 Chapter 8, 14 Films: <u>Surface Area of Solid I.</u> (CEF) Color, 15 min. <u>Surface Area of Solid II.</u> (CEF) Color, 15 min. <u>Volumes of Cubes, Prisms, and Cylinders.</u> (CEF) Color, 30 min. <u>Volumes of Pyramids, Cones, and Spheres.</u> (CEF) Color, 15 min.
For any finite region and any line whose intersection with the region is not empty, the union of the points of intersection have a greatest lower bound and least upper bound.	
Stress should be placed upon the fact that similarity implies proportionality of corresponding segments. Therefore, areas, which are basically a product of two perpendicular segments, will be proportional to the squares of the measures of one-dimensional elements. Following the same reasoning, volumes will be proportioned to the cubes of the measures of one-dimensional elements. While formulas can state the proportionality between volumes, it would be well to avoid giving the student the formulas until ample discovery exercises have been offered. Development might start with the cube and right rectangular prism and continue into other solids as his comprehension grows.	

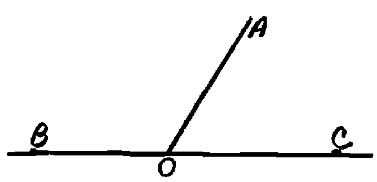
SUGGESTED EVALUATIVE TEST ITEMS

The test items included in this guide are considered examples of some criteria to be used to determine whether outcomes have or have not been achieved. Some might be used for diagnosis of learning difficulties. The major reason for their inclusion was to place emphasis upon important outcomes.

The items may be incorporated in tests of major areas or units in the form shown or in alternate forms. Possibly they could be used as part of semester or final examinations. They could be suggestive of items for possible city-wide surveys of achievement in geometry. By no means should these items be considered as covering every facet of evaluation of achievement in geometry.

* *

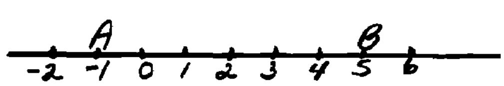
1.-I.



In the drawing, if BOC is a straight, line ∠AOB and ∠AOC can be described as:

a) adjacent and supplementary
b) equal
c) adjacent and complementary
d) complementary and vertical
e) none of these

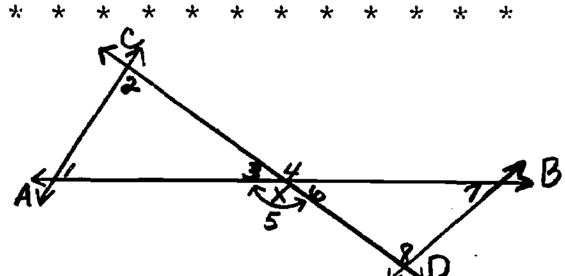
2.-I.



The length of $\overline{AB}$ is:

a) -6
b) 4
c) 6
d) -4
e) none of these

3.-I.



From the drawing, select the correct statements:

I -- ∠3 and ∠6 are vertical
II -- ∠5 and ∠4 are supplementary
III -- Ray XD and Ray XC are opposite Rays

a) I and II are correct
b) II and III are correct
c) I and III are correct
d) all three statements are correct
e) all three statements are not correct

4.-I.

A on a number has coordinate - 1
B on the same number line has coordinate - 5
The length of $\overline{AB}$ is:

a) -6
b) 4
c) 6
d) -4
e) none of these

5.-I.

If set x contains 6 elements,
set y contains 10 elements,
and $x \cap y$ contains 3 elements,
then $x \cup y$ contains:

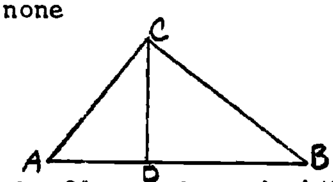
a) 13
b) 16
c) 19
d) 7
e) none of these

6.-I.

If a point X is between the points R and S, which of the following must be true?

a) RX = SX
b) RS + SX = RX
c) R + RS = S
d) RX + XS = RS
e) none

7.-II.



Using the figure shown in ΔABC, $\overline{CD}$ is perpendicular to $\overline{AB}$. Which of the following must be true?

a) $\overline{CD}$ bisects $\overline{AB}$
b) $\overline{CD}$ is an altitude
c) $\overline{CD}$ is a median
d) $\overline{CD}$ bisects $\angle ACB$
e) none of these

8.-II.

If Joe goes to church every Sunday and today is Sunday, we can conclude Joe will go to church today. This is an example of:

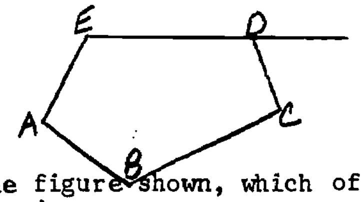
a) inductive reasoning
b) indirect reasoning
c) deductive reasoning
d) intuition
e) none of these

9.-II.

A statement accepted without proof is called:

a) a postulate
b) a theorem
c) a corollary
d) a converse
e) none of these

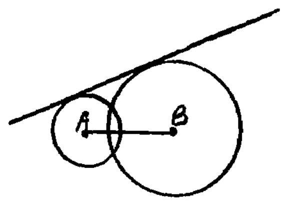
10.-II.



Using the figure shown, which of the following is not a correct name for the polygon?

a) ABCDE
b) DEABC
c) BCAED
d) AEDCB
e) CBAED

11.-II.



Using the figure shown, circles A and B:

a) have common centers
b) have a common internal tangent
c) have a common chord
d) have common radii
e) have none of these

12.-II.

Given that 3x = 12, select the property which proves that x = 4:

a) transitive property of real numbers
b) subtraction property of real numbers
c) division property of real numbers
d) symmetric property of real numbers
e) addition property of real numbers

13.-III.

Given $\angle 1$ and $\angle 3$ are complementary
$\angle 1$ and $\angle 2$ are supplementary

I $\angle 2 = \angle 3$
II $\angle 2$ is obtuse
III $\angle 2 = \angle 3 + 90$

Select the correct answer:

a) I and II are true
b) II and III are true
c) only I is true
d) only III is true
e) none of the statements are true

14.-III.

Given: $\angle a$ and $\angle b$ are supplementary and $\angle c$ and $\angle b$ are supplementary. Which of these statements must be true?

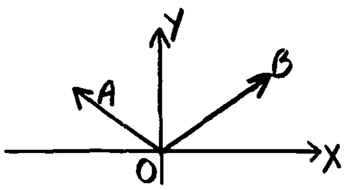
a) $\angle a$ and $\angle c$ are supplementary
b) $\angle a$ is a right angle
c) $\angle a = \angle c$
d) $\angle a$ and $\angle b$ are adjacent
e) none of the above are true

15.-III.

The supplement of an angle is three times its complement. The measure of the angle is:

a) 60
b) 30
c) 270
d) no such angle
e) 45

16.-III.



Given $\overrightarrow{OX} \perp \overrightarrow{OY}$ and $\overrightarrow{OA} \perp \overrightarrow{OB}$, $\overrightarrow{OB}$ bisects $\angle YOX$. Using the figure, the measure of $\angle AOX$ is:

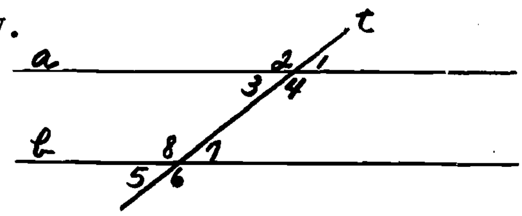
a) 120
b) 130
c) 90
d) 45
e) 135

17.-III.

If ray AM lies between ray AC and ray AD in a half-plane, which of the following must be true:

a) $\angle MAC = \angle MAD + \angle DAC$
b) $\angle CAD = \angle CAM + \angle MAD$
c) ray AM bisects $\angle CAD$
d) AC and AD are opposite rays
e) none of the above are true

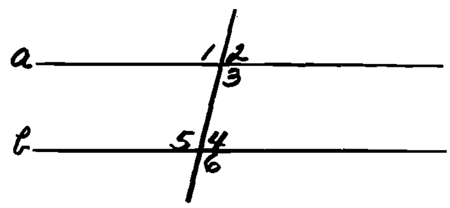
18.-IV.



Line a || line b cut by transversal t. The measure of $\angle 1 = (x-15)$. From these facts the correct measure for $\angle 8$ is:

a) $(90 - x)$
b) $(180 - x)$
c) $(165 - x)$
d) 105
e) $(195 - x)$

19.-IV.



Using the figure shown, a || b, which of the following pairs of angles need not be equal?

a) $\angle 1$ and $\angle 6$
b) $\angle 3$ and $\angle 4$
c) $\angle 1$ and $\angle 5$
d) $\angle 3$ and $\angle 5$
e) $\angle 5$ and $\angle 6$

20.-IV.

The measures of the angles of a triangle are in the ratio of 1:3:5. The smallest angle has a measure of:

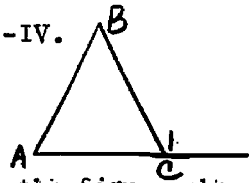
a) 10
b) 1
c) 30
d) 20
e) none of these

21.-IV.

$\overline{AB}$, $\overline{CD}$, and $\overline{XY}$ are co-planar and $\overline{AB} \perp \overline{XY}$ at X and $\overline{CD} \perp \overline{XY}$ at Y. Which statement must be true?

a) $\overline{AB} \parallel \overline{CD}$
b) $\overline{AB} \perp \overline{CD}$
c) $\overline{AB} = \overline{CD}$
d) $AB = CD$
e) none of these are true

22.-IV.



In the figure shown, $\angle 1 = m^\circ$ and $\angle A = x^\circ$. The measure of $\angle B$ is:

a) $m^\circ + x^\circ$
b) $x^\circ - m^\circ$
c) $m^\circ - x^\circ$
d) $180 - m^\circ$
e) $\frac{m + x}{2}$

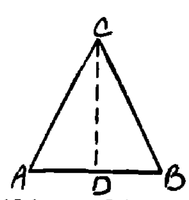
23.-V.

I. SAS
II. SSS
III. SSA
IV. AAS
V. AAA

Which of the above correspondences are not acceptable for triangle congruence:

a) I and III
b) II and IV
c) III and V
d) IV and I
e) II and V

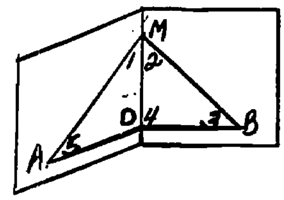
24.-V.



An auxiliary line is drawn from vertex C of ΔABC and intersects $\overline{AB}$ at point D. Which of the following conditions, in general, would be improper:

a) $\overrightarrow{CD}$ bisects $\angle C$
b) $\overline{CD}$ is a median of ΔABC
c) $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$
d) $AD:DB = 2:3$
e) $\overline{CD} \perp \overline{AB}$

25.-V.



Using the drawing to the right, $\overline{AD}$ and $\overline{BD}$ are perpendicular to $\overline{MD}$ at D. If $\overline{BD} = \overline{AD}$, which statement must be true:

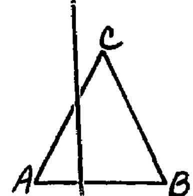
a) $\angle 4 = \angle 3$
b) $\angle 1 = \angle 5$
c) $\angle 1 = \angle 3$
d) $\angle 1 = \angle 2$ and $\overline{AM} = \overline{MD}$
e) $\angle 1 = \angle 2$ and $\overline{AM} = \overline{MB}$

26.-V.

The diagonals of a rectangle:

a) are perpendicular
b) bisect the angles of the rectangle
c) are equal in length
d) are parallel
e) none of these

27.-V.



Given AC = BC and A = 80, the measure of $\angle C$ is:

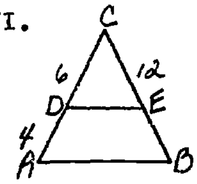
a) 80
b) 100
c) 20
d) 50
e) none of these

28.-V.

In parallelogram ABCD, $\angle A = 80^\circ$. The measure of $\angle B$ is:

a) 100
b) 50
c) 10
d) 80
e) none of these

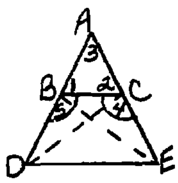
29.-VI.



In the figure shown, $\overline{DE} \parallel \overline{AB}$ and segments have the measures shown. The measure of $\overline{BE}$ is:

a) 18
b) 2
c) 10
d) 8
e) none of these

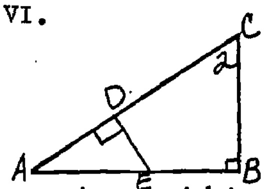
30.-VI.



BC parallel to DE and makes ΔABC equilateral. The statement which must be true:

a) AB = BD
b) BD $\neq$ CE
c) BC = BD
d) $\Delta BCE \cong \Delta CBD$
e) B is midpoint of AD

31.-II. or VI.



In the figure shown, which of the following statements must be true?

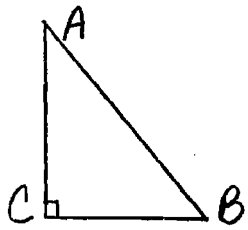
a) $\overline{DE} = \overline{BC}$
b) $\angle 1 = \angle 2$
c) $\angle 1 + \angle 2 = 180$
d) E is the midpoint of $\overline{AB}$
e) none of the above must be true

32.-VI.

In triangle ABC, $\angle C = 90^\circ$, AC = 6 and BC = 8. The measure of AB:

a) is 10
b) is 14
c) is 7
d) is $\sqrt{14}$
e) is not given above

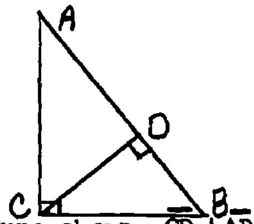
33.-VI.



In the figure shown, $\angle A = 30^\circ$, $\angle C = 90^\circ$, and AB = 6. Which of the following statements are true?

a) AC = 3
b) BC = 12
c) AC = 6
d) BC = 3
e) BC = $3\sqrt{3}$

34.-VI.



In the figure shown, $\overline{CD} \perp \overline{AB}$ and $\overline{AC} \perp \overline{CB}$. If AD = 9 and DB = 4, the measure of CD is:

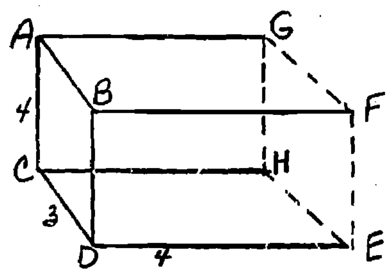
a) $6\frac{1}{2}$
b) greater than 9
c) less than 4
d) 7
e) 6

35.-VI.

If $\frac{a}{b} = \frac{c}{d}$, then :

a) ad = bc
b) ac = bd
c) ab = cd
d) a + b = c + d
e) a - b = c - d

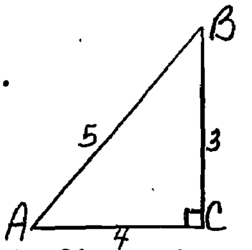
36.-VI.



The figure represents a rectangular box with dimensions shown. Find the length of the diagonal AE:

a) 5
b) $\sqrt{32}$
c) $\sqrt{22}$
d) $\sqrt{41}$
e) 11

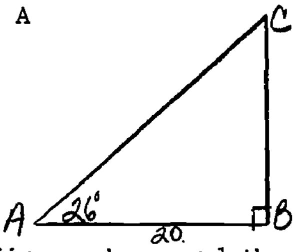
37.-VII.



Using the figure shown, $\frac{3}{5}$ would be the value of:

a) tan A
b) tan B
c) sin C
d) sin A
e) cos A

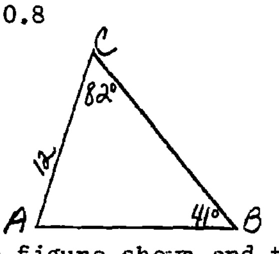
38.-VII.



Use the figure shown and the ratios,
$\sin 26^\circ = .44$
$\cos 26^\circ = .90$
$\tan 26^\circ = .49$
Which of the following would be the closest approximation to BC?

a) 8.8
b) 9.8
c) 18.0
d) 22.2
e) 40.8

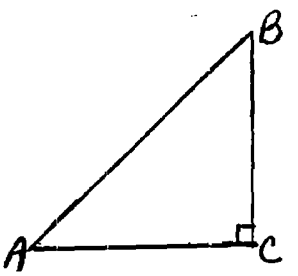
39.-VII.



Use the figure shown and the ratios:
$\sin 41^\circ = .66$
$\sin 82^\circ = .99$
Which of the following would be the best approximation for AB?

a) 8
b) 12
c) 18
d) 20
e) 24

40.-VII.



Sin A=

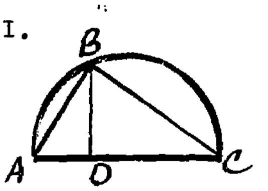
a) $\frac{AC}{BC}$
b) $\frac{BC}{AC}$
c) $\frac{AC}{AB}$
d) $\frac{BC}{AB}$
e) $\frac{AB}{AC}$

41.-VII.

Sin x $<$ cos x if the measure of angle x is:

a) $0 < x \leq 45$
b) $0 \leq x < 45$
c) $0 < x < 90$
d) $45 \leq x < 90$
e) $45 < x \leq 90$

42.-VIII.



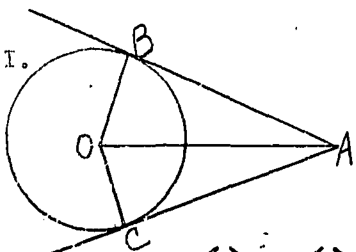
Given semi-circle ABC, with $\overline{BD}$ perpendicular to the diameter $\overline{AC}$,

I-Triangle ABC is an isoceles triangle
II-Triangle ADB is similar to $\triangle$ABC
III-$BD^2 = (AD) \times DC$ or $\frac{AD}{BD} = \frac{BD}{DC}$
IV-$AC^2 = AB^2 + BD^2 + CB^2$

Select the correct statement:

a) I only is true
b) I and II are true
c) II and III are true
d) III only is true
e) II and IV are true

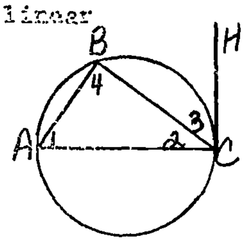
43.-VIII.



In the figure shown $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are tangents to circle O at points B and C, respectively. Which one of the following statements are not true?

a) $\overline{AO}$ bisects $\angle BAC$
b) AB = AC
c) $\triangle AOB$ is a right triangle
d) $\overline{AO}$ bisects $\widehat{BC}$
e) points B, O, and C might be collinear

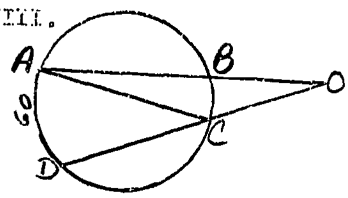
44.-VIII.



AC is diameter of the circle and points A, B, and C are on the circle. Angle 1 has a measure of 55. Using the figure, choose the only correct true statement:

a) $\angle 3$ = 35 and equal to $\widehat{AB}$
b) $\angle 1 = \angle 2$ and $\triangle ABC$ is isosceles
c) $\angle 4$ = 90 and $\angle 2$ = 15
d) $\angle 4$ = 90 and measure of $\widehat{AB}$ = 70
e) All angles are acute

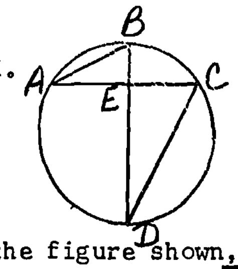
45.-VIII.



Using the figure shown, the measure of $\angle O$ is 15. The measure of $\widehat{CB}$ is:

a) 25
b) 32
c) 28
d) 30
e) none of these

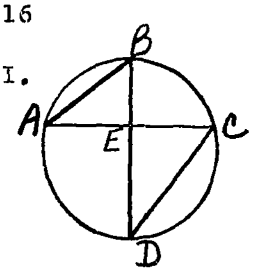
46.-VIII.



Using the figure shown, AE=6, EC=8, aand BE=4. The measure of $\overline{ED}$ is:

a) 3
b) 5 2/3
c) 10
d) 12
e) 16

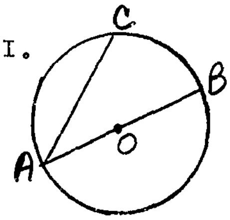
47.-VIII.



Using the figure shown, $\angle B$ = 70 . The measure of $\angle C$ is:

a) 140
b) 35
c) 110
d) 20
e) none of the above

48.-VIII.



In the figure shown, AB is a diameter of circle O and $\widehat{AC}$ = 100 . The measure of $\angle CAB$ is:

a) 40
b) 50
c) 80
d) 100
e) none of the above

49.-VIII.

In a circle, chord AB is nearer to the center than chord CD. Which of the following statements must be true?

a) AB = CD
b) AB > CD
c) AB < CD
d) $\overline{AB} \parallel \overline{CD}$
e) no conclusion can be drawn

50.-VIII.

In a circle, chord AB is 6" long and is 4" from the center of the circle. The radius of the circle is:

a) 2"
b) 4"
c) 5"
d) 6"
e) 10"

51.-IX.

The construction method for circumscribing a circle about a scalene triangle requires finding its center by constructing the:

a) perpendicular bisector of the sides
b) bisectors of the angles
c) medians of the triangle
d) altitudes of the triangle
e) parallel lines bisecting two sides of the triangle

52.-IX.

The locus of points lying in a given plane and equidistant from points A and B in the plane is:

a) a circle with $\overline{AB}$ as diameter
b) a circle with A as center and $\overline{AB}$ as radius
c) the midpoint of $\overline{AB}$
d) a segment that is the perpendicular bisector of $\overline{AB}$
e) the line that is the perpendicular bisector of $\overline{AB}$

53.-IX.

The locus of points in space that are 5" from point A is:

a) a circle with A as center and a 5" radius
b) two parallel planes 5" on either side of A
c) the surface of a sphere with A as center and a 5" radius
d) two spheres tangent at A and with 5" radii
e) none of the above

54.-IX.

Which of the following are permissible ___in making a construction:

I: ruler
II: straight-edge
III: protractor
IV: compass

a) I and II
b) I and III
c) I and IV
d) II and III
e) II and IV

55.-X.

A line segment $\overline{OP}$ is drawn from the point (10, 0) to the point 6, 4). What are the coordinates of the midpoint of $\overline{OP}$?

a) (6, 8)
b) (8, 2)
c) (8, 4)
d) (16, 2)
e) (12, 8)

56.-X.

A triangle which has vertices (0, 0), (2, 3), and (3, -2) belongs to which of the following sets:

I - Isosceles triangles
II - Right triangles
III - Equilateral triangles

a) none of these
b) I only
c) II only
d) III only
e) I and II only

57.-X.

On the coordinate plane, three vertices of a rectangle have coordinates of (0, 0), (a, 0), and (0, b), respectively. The fourth vertex will be:

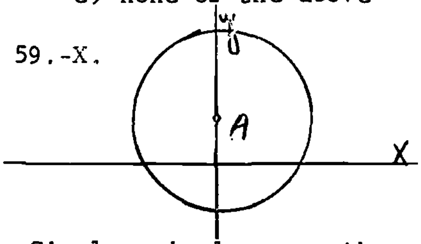
a) (0, a)
b) (b, 0)
c) (a, b)
d) (a + b, a + b)
e) none of the above

58.-X.

(3, 6) and (-2, 3) are two points on the coordinate plane. The distance between these points is:

a) $\sqrt{34}$
b) $\sqrt{10}$
c) 8
d) 4
e) none of the above

59.-X.



Circle A is drawn on the coordinate plane. Circle A is symmetric with respect to:

a) the x-axis, the y-axis, and the origin
b) the x-axis, the y-axis and point A
c) the x-axis and point A
d) the y-axis and point A
e) the y-axis and the origin

60.-X.

Given the following points on a coordinate plane:

I. - (3, -2)
II. - (-2, -3)
III. - (-4, 0)
IV. - (2, -2)
V. - (-3, 0)

The points which are in the fourth quadrant are:

a) I and II
b) III and IV
c) III and V
d) I and IV
e) I, III, IV, and V

61.-X.

On the coordinate plane, a line is drawn with slope $\frac{1}{2}$ through the point (1, -2). An equation for this line is:

a) $y + 2 = \frac{1}{2}(x-1)$
b) $y - 2 = 2(x-1)$
c) $y - 2 = \frac{1}{2}(x+1)$
d) $y - 1 = \frac{1}{2}(x+2)$
e) none of the above

62.-X.

Two lines on the coordinate plane are perpendicular. If the slope of one line is $\frac{2}{3}$, the slope of the other is:

a) $-\frac{2}{3}$

b) $-\frac{3}{2}$

c) $\frac{2}{3}$

d) $-\frac{3}{2}$

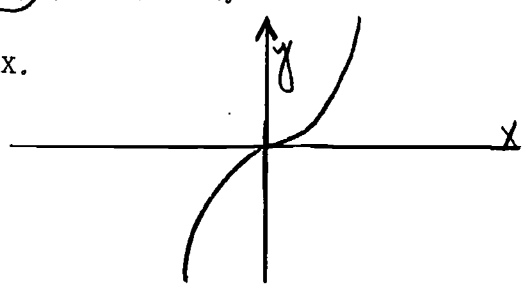
e) none of the above

63.-X.

On the coordinate plane, a circle is drawn with center at (3, 2) and radius of 5". The equation of the circle is:

a) $3x + 2y = 5$
b) $(x + 3)^2 + (y + 2)^2 = 25$
c) $(x - 2)^2 + (y - 3)^2 = 25$
d) $(x - 3)^2 - (y - 2)^2 = 25$
e) $(x - 3)^2 + (y - 2)^2 = 25$

64.-X.

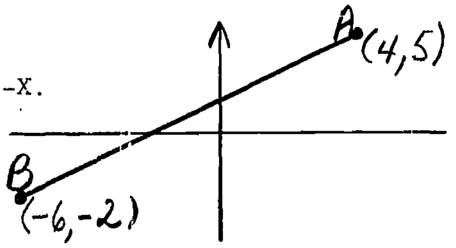


Using the figure, the best fitting statement below is:

The curve is symmetrical to:

a) the x-axis
b) the y-axis
c) a point on x-axis
d) a point on the y-axis
e) the origin

65.-X.



Using the figure shown, the only true statement is:

The coordinates of the midpoint of $\overline{BA}$ are:

a) $(-1, -\frac{3}{2})$

b) $(-1, -\frac{1}{2})$

c) $(-1, -1)$

d) $(-2, \frac{3}{2})$

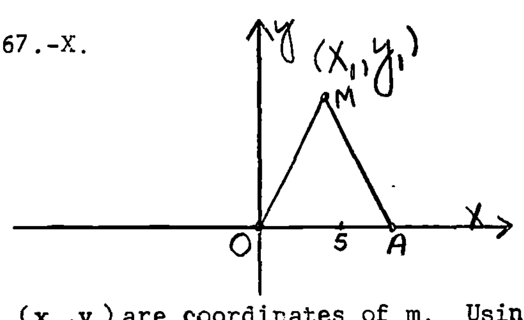
e) $(-1, \frac{3}{2})$

66.-X.

Given that $-4 \geq x$ and $4 \leq y$, the whole numbers excluded from the union of these sets are:

a) -4, -3, -2, -1, 0, 1, 2, 3
b) -3, -2, -1, 0, 1, 2, 3, 4
c) -3, -2, -1, 0, +1, +2, +3
d) -16, -12, -3, -5, -1
e) none of these

67.-X.



(x_1, y_1) are coordinates of m. Using the figure above, the only true statement below is:

a) the measure of OM = $\sqrt{x_1^2 + y_1^2 + 5}$

b) if m moves at constant distance from O its equation is $x_1^2 + y_1^2 = \overline{OM}^2$

c) $OM^2 = \sqrt{AM}$

d) $\angle MOA$ must equal $\angle MAO$

e) none of these

68.-XI.

A rectangle has sides of 8" and 6". Which of the following measures would give a right triangle an area equal to that of the rectangle?

a) legs of 3" and 4"
b) legs of 8" and 12"
c) legs of 16" and 12"
d) a leg of 8" and a hypotenuse of 12"
e) legs of 8" and 16"

69.-XI.

The shortest sides of two similar triangles are 4" and 8", respectively. If the area of the larger triangle is 36 sq. in., the area of the smaller is:

a) 18 sq. in.
b) 72 sq. in.
c) 9 sq. in.
d) 144 sq. in.
e) none of the above

70.-XI.

A circle has radius of 12". An arc of the circle has measure of 120. The length of the arc is:

a) 24π inches
b) 12π inches
c) 6 inches
d) 8π inches
e) 24 inches

71.-XI.

The exact ratio of the circumference of a circle to its diameter is:

a) $\frac{22}{7}$

b) π

c) 3.1415926535
d) 3.1416
e) 2

72.-XI.

A parallelogram has an angle of 30° and sides of 10" and 8" Its area is:

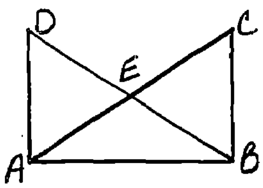
a) 80 sq. in.
b) $40\sqrt{2}$ sq. in.
c) $40\sqrt{3}$ sq. in.
d) $80\sqrt{3}$ sq. in.
e) 40 sq. in.

73.-XI.

The diagonals of a rhombus are 10" and 6" in length. Its area is:

a) 120 sq. in.
b) 60 sq. in.
c) 30 sq. in.
d) 16 sq. in.
e) 8 sq. in.

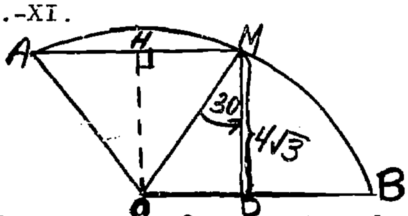
74.-XI.



In the figure shown, the area of $\triangle$ABD is 12, the area of $\triangle$ABC is 12, and the area of $\triangle$ABE is 7. The area of the polygonal region ABCED is:

a) 31
b) 24
c) 19
d) 17
e) none of the above

75.-XI.



The measure of central angle AOB is 120. M is midpoint of $\widehat{AMB}$ and is endpoint of radius OM. Using the figure and dimensions shown, the area of $\triangle$AOM is:

a) equal to $\frac{1}{2}$ $\overline{AMDO}$
b) equal to $\overline{AM}$ X $\overline{MD}$
c) equal to OHMD
d) equal to 2 X $4\sqrt{3}$
e) none of these

76.-XI.

A circle has area of 620 square units. The area of the circle having a radius one-half that of the given circle is:

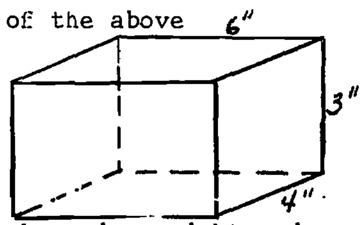
a) 320 sq. in.
b) 420 sq. in.
c) 1240 sq. in.
d) 155 sq. in.
e) none of these

77.-XII.

Two solids are similar. The larger has a volume of 54 cu. in. and a side of 6". If the corresponding side of the smaller is 4", the volume of the smaller is:

a) 16 cu. in.
b) 36 cu. in.
c) 24 cu. in.
d) 30 cu. in.
e) none of the above

78.-XII.



The figure shown is a right prism with a rectangular base. Its volume is:

a) 13 cu. in.
b) 72 in.
c) 72 sq. in.
d) 72 cu. in.
e) 30 sq. in.

79.-XII.

A cube has an edge of 5". Its total area is:

a) 40 sq. in.
b) 25 sq. in.
c) 100 sq. in.
d) 125 sq. in.
e) 150 sq. in.

80.-XII.

A right circular cylinder has a radius of 3" and an altitude of 5". Its volume is:

a) 15 π cu. in.
b) 45 π cu. in.
c) 30 π cu. in.
d) 75 π cu. in.
e) none of the above

81.-XII.

If only planes are used to form the surface of a solid, the minimum number needed is:

a) 2
b) 3
c) 4
d) 5
e) 6

BIBLIOGRAPHY FOR GEOMETRY

1. Goodwin, Wilson A., and others. Geometry: A Unified Course. Columbus, Ohio: Charles E. Merrill Books, Inc., 1967.

2. Hart, Walter W., and others. New Plane Geometry and Supplements. Boston, Massachusetts: D. C. Heath and Company, 1964.

3. Henderson, Kenneth B., and others. Modern Geometry: Its Structure and Function. Chicago, Illinois: McGraw-Hill, 1962.

4. Jurgensen, Ray C., and others. Modern Geometry: Structure and Method. Geneva, Illinois: Houghton Mifflin Company, 1965.

5. Moise, Edwin E., and Floyd L. Downs. Geometry. Reading, Massachusetts: Addison-Wesley Publishing Company, 1964.

6. Schacht, J. F., and others. Contemporary Geometry. New York, New York: Holt, Rinehart, and Winston, Inc., 1962.

7. Smith, R. R. and J. F. Ulrich. Geometry: A Modern Course. New York, New York: Harcourt, Brace, and World, Inc., 1964.

8. Welchons, A. M. Plane Geometry. New York, New York: Ginn and Company, 1958.

9. Wichura, Gerhard. Geometry. Reading, Massachusetts: Addison-Wesley Publishing Company, 1958.

10. School Mathematics Study Group. Geometry with Coordinates, Part I. New Haven, Connecticut: Yale University Press, 1962.

11. School Mathematics Study Group. Geometry with Coordinates, Part II. New Haven, Connecticut: Yale University Press, 1962.